Part V
Abstraction

We've really been talking about abstraction all along. Whenever you find yourself performing several similar computations, such as

```
> (sentence 'she (word 'run 's))
(SHE RUNS)

> (sentence 'she (word 'walk 's))
(SHE WALKS)

> (sentence 'she (word 'program 's))
(SHE PROGRAMS)
```

and you capture the similarity in a procedure

```
(define (third-person verb)
  (sentence 'she (word verb 's)))
```

you're *abstracting* the pattern of the computation by expressing it in a form that leaves out the particular verb in any one instance.

In the preface we said that our approach to computer science is to teach you to think in larger chunks, so that you can fit larger problems in your mind at once; "abstraction" is the technical name for that chunking process.

In this part of the book we take a closer look at two specific kinds of abstraction. One is *data abstraction,* which means the invention of new data types. The other is the implementation of *higher-order functions,* an important category of the same process abstraction of which `third-person` is a trivial example.

Until now we've used words and sentences as though they were part of the natural order of things. Now we'll discover that Scheme sentences exist only in our minds and take shape through the use of constructors and selectors (`sentence`, `first`, and so on) that we wrote. The implementation of sentences is based on a more fundamental data type called *lists*. Then we'll see how lists can be used to invent another in-our-minds data type, *trees*. (The technical term for an invented data type is an *abstract* data type.)

You already know how higher-order functions can express many computational processes in a very compact form. Now we focus our attention on the higher-order *procedures* that implement those functions, exploring the mechanics by which we create these process abstractions.


NICE
Aubrey
Gerry
Guy
John
Marc
NAUGHTY
Bjarne
Edsger
Niklaus

17 Lists

Suppose we're using Scheme to model an ice cream shop. We'll certainly need to know all the flavors that are available:

```
(vanilla ginger strawberry lychee raspberry mocha)
```

For example, here's a procedure that models the behavior of the salesperson when you place an order:

```
(define (order flavor)
  (if (member? flavor
               '(vanilla ginger strawberry lychee raspberry mocha))
      '(coming right up!)
      (se '(sorry we have no) flavor)))
```

But what happens if we want to sell a flavor like "root beer fudge ripple" or "ultra chocolate"? We can't just put those words into a sentence of flavors, or our program will think that each word is a separate flavor. Beer ice cream doesn't sound very appealing.

What we need is a way to express a collection of items, each of which is itself a collection, like this:

```
(vanilla (ultra chocolate) (heath bar crunch) ginger (cherry garcia))
```

This is meant to represent five flavors, two of which are named by single words, and the other three of which are named by sentences.

Luckily for us, Scheme provides exactly this capability. The data structure we're using in this example is called a *list*. The difference between a sentence and a list is that the elements of a sentence must be words, whereas the elements of a list can be anything

at all: words, `#t`, procedures, or other lists. (A list that's an element of another list is called a *sublist*. We'll use the name *structured* list for a list that includes sublists.)

Another way to think about the difference between sentences and lists is that the definition of "list" is self-referential, because a list can include lists as elements. The definition of "sentence" is not self-referential, because the elements of a sentence must be words. We'll see that the self-referential nature of recursive procedures is vitally important in coping with lists.

Another example in which lists could be helpful is the pattern matcher. We used sentences to hold `known-values` databases, such as this one:

```
(FRONT YOUR MOTHER ! BACK SHOULD KNOW !)
```

This would be both easier for you to read and easier for programs to manipulate if we used list structure to indicate the grouping instead of exclamation points:

```
((FRONT (YOUR MOTHER)) (BACK (SHOULD KNOW)))
```

We remarked when we introduced sentences that they're a feature we added to Scheme just for the sake of this book. Lists, by contrast, are at the core of what Lisp has been about from its beginning. (In fact the name "Lisp" stands for "LISt Processing.")

Selectors and Constructors

When we introduced words and sentences we had to provide ways to take them apart, such as `first`, and ways to put them together, such as `sentence`. Now we'll tell you about the selectors and constructors for lists.

The function to select the first element of a list is called `car`.* The function to select the portion of a list containing all but the first element is called `cdr`, which is

* Don't even try to figure out a sensible reason for this name. It's a leftover bit of history from the first computer on which Lisp was implemented. It stands for "contents of address register" (at least that's what all the books say, although it's really the address *portion* of the accumulator register). `Cdr`, coming up in the next sentence, stands for "contents of decrement register." The names seem silly in the Lisp context, but that's because the Lisp people used these register components in ways the computer designers didn't intend. Anyway, this is all very interesting to history buffs but irrelevant to our purposes. We're just showing off that one of us is actually old enough to remember these antique computers first-hand.

pronounced "could-er." These are analogous to `first` and `butfirst` for words and sentences.

Of course, we can't extract pieces of a list that's empty, so we need a predicate that will check for an empty list. It's called `null?` and it returns `#t` for the empty list, `#f` for anything else. This is the list equivalent of `empty?` for words and sentences.

There are two constructors for lists. The function `list` takes any number of arguments and returns a list with those arguments as its elements.

```
> (list (+ 2 3) 'squash (= 2 2) (list 4 5) remainder 'zucchini)
(5 SQUASH #T (4 5) #<PROCEDURE> ZUCCHINI)
```

The other constructor, `cons`, is used when you already have a list and you want to add one new element. `Cons` takes two arguments, an element and a list (in that order), and returns a new list whose `car` is the first argument and whose `cdr` is the second.

```
> (cons 'for '(no one))
(FOR NO ONE)

> (cons 'julia '())
(JULIA)
```

There is also a function that combines the elements of two or more lists into a larger list:

```
> (append '(get back) '(the word))
(GET BACK THE WORD)
```

It's important that you understand how `list`, `cons`, and `append` differ from each other:

```
> (list '(i am) '(the walrus))
((I AM) (THE WALRUS))

> (cons '(i am) '(the walrus))
((I AM) THE WALRUS)

> (append '(i am) '(the walrus))
(I AM THE WALRUS)
```

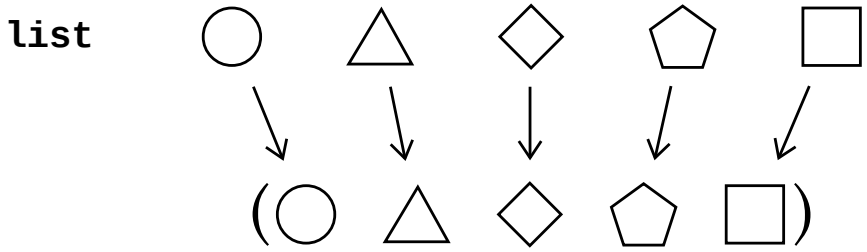
When `list` is invoked with two arguments, it considers them to be two proposed elements for a new two-element list. `List` doesn't care whether the arguments are themselves lists, words, or anything else; it just creates a new list whose elements are the arguments. In this case, it ends up with a list of two lists.

`Cons` requires that its second argument be a list.* `Cons` will extend that list to form a new list, one element longer than the original; the first element of the resulting list comes from the first argument to `cons`. In other words, when you pass `cons` two arguments, you get back a list whose `car` is the first argument to `cons` and whose `cdr` is the second argument.

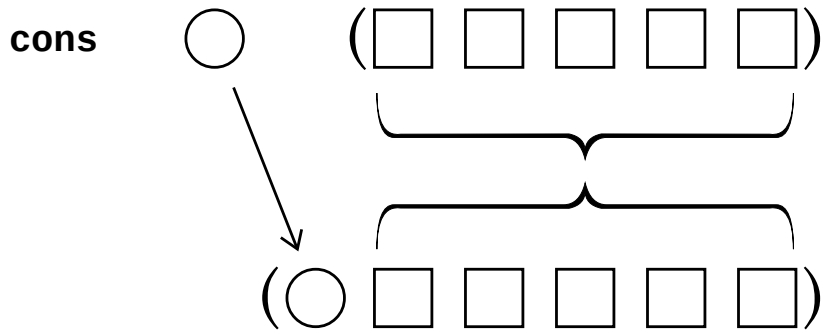
Thus, in this example, the three elements of the returned list consist of the first argument as one single element, followed by *the elements of* the second argument (in this case, two words). (You may be wondering why anyone would want to use such a strange constructor instead of `list`. The answer has to do with recursive procedures, but hang on for a few paragraphs and we'll show you an example, which will help more than any explanation we could give in English.)

Finally, `append` of two arguments uses the elements of *both* arguments as elements of its return value.

Pictorially, `list` creates a list whose elements are the arguments:

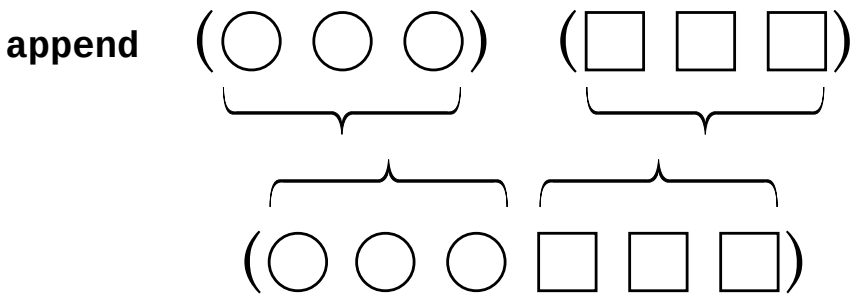


`Cons` creates an extension of its second argument with one new element:



* This is not the whole story. See the "pitfalls" section for a slightly expanded version.

`Append` creates a list whose elements are the *elements of* the arguments, which must be lists:



Programming with Lists

```
(define (praise flavors)
  (if (null? flavors)
      '()
      (cons (se (car flavors) '(is delicious))
            (praise (cdr flavors)))))

> (praise '(ginger (ultra chocolate) lychee (rum raisin)))
((GINGER IS DELICIOUS) (ULTRA CHOCOLATE IS DELICIOUS)
 (LYCHEE IS DELICIOUS) (RUM RAISIN IS DELICIOUS))
```

In this example our result is a *list of sentences*. That is, the result is a list that includes smaller lists as elements, but each of these smaller lists is a sentence, in which only words are allowed. That's why we used the constructor `cons` for the overall list, but `se` for each sentence within the list.

This is the example worth a thousand words that we promised, to show why `cons` is useful. `List` wouldn't work in this situation. You can use `list` only when you know exactly how many elements will be in your complete list. Here, we are writing a procedure that works for any number of elements, so we recursively build up the list, one element at a time.

In the following example we take advantage of structured lists to produce a translation dictionary. The entire dictionary is a list; each element of the dictionary, a single translation, is a two-element list; and in some cases a translation may involve a phrase rather than a single word, so we can get three deep in lists.

```
(define (translate wd)
  (lookup wd '((window fenetre) (book livre) (computer ordinateur)
               (house maison) (closed ferme) (pate pate) (liver foie)
               (faith foi) (weekend (fin de semaine))
               ((practical joke) attrape) (pal copain))))

(define (lookup wd dictionary)
  (cond ((null? dictionary) '(parlez-vous anglais?))
        ((equal? wd (car (car dictionary)))
         (car (cdr (car dictionary))))
        (else (lookup wd (cdr dictionary)))))

> (translate 'computer)
ORDINATEUR

> (translate '(practical joke))
ATTRAPE

> (translate 'recursion)
(PARLEZ-VOUS ANGLAIS?)
```

By the way, this example will help us explain why those ridiculous names `car` and `cdr` haven't died out. In this not-so-hard program we find ourselves saying

```
(car (cdr (car dictionary)))
```

to refer to the French part of the first translation in the dictionary. Let's go through that slowly. `(Car dictionary)` gives us the first element of the dictionary, one English-French pairing. `Cdr` of that first element is a one-element list, that is, all but the English word that's the first element of the pairing. What we want isn't the one-element list but rather its only element, the French word, which is its `car`.

This `car` of `cdr` of `car` business is pretty lengthy and awkward. But Scheme gives us a way to say it succinctly:

```
(cadar dictionary)
```

In general, we're allowed to use names like `cddadr` up to four deep in `A`s and `D`s. That one means

```
(cdr (cdr (car (cdr something))))
```

or in other words, take the `cdr` of the `cdr` of the `car` of the `cdr` of its argument. Notice that the order of letters `A` and `D` follows the order in which you'd write the procedure names, but (as always) the procedure that's invoked first is the one on the right. Don't make the mistake of reading `cadr` as meaning "first take the `car` and then take the `cdr`." It means "take the `car` of the `cdr`."

The most commonly used of these abbreviations are `cadr`, which selects the second element of a list; `caddr`, which selects the third element; and `cadddr`, which selects the fourth.

The Truth about Sentences

You've probably noticed that it's hard to distinguish between a sentence (which *must* be made up of words) and a list that *happens* to have words as its elements.

The fact is, sentences *are* lists. You could take `car` of a sentence, for example, and it'd work fine. Sentences are an abstract data type represented by lists. We created the sentence ADT by writing special selectors and constructors that provide a different way of using the same underlying machinery—a different interface, a different metaphor, a different point of view.

How does our sentence point of view differ from the built-in Scheme point of view using lists? There are three differences:

- A sentence can contain only words, not sublists.
- Sentence selectors are symmetrical front-to-back.
- Sentences and words have the same selectors.

All of these differences fit a common theme: Words and sentences are meant to represent English text. The three differences reflect three characteristics of English text: First, text is made of sequences of words, not complicated structures with sublists. Second, in manipulating text (for example, finding the plural of a noun) we need to look at the end of a word or sentence as often as at the beginning. Third, since words and sentences work together so closely, it makes sense to use the same tools with both. By contrast, from Scheme's ordinary point of view, an English sentence is just one particular case of a much more general data structure, whereas a symbol* is something entirely different.

* As we said in Chapter 5, "symbol" is the official name for words that are neither strings nor numbers.

The constructors and selectors for sentences reflect these three differences. For example, it so happens that Scheme represents lists in a way that makes it easy to find the first element, but harder to find the last one. That's reflected in the fact that there are no primitive selectors for lists equivalent to `last` and `butlast` for sentences. But we want `last` and `butlast` to be a part of the sentence package, so we have to write them in terms of the "real" Scheme list selectors. (In the versions presented here, we are ignoring the issue of applying the selectors to words.)

```
(define (first sent)                          ;;; just for sentences
  (car sent))

(define (last sent)
  (if (null? (cdr sent))
      (car sent)
      (last (cdr sent))))

(define (butfirst sent)
  (cdr sent))

(define (butlast sent)
  (if (null? (cdr sent))
      '()
      (cons (car sent) (butlast (cdr sent)))))
```

If you look "behind the curtain" at the implementation, `last` is a lot more complicated than `first`. But from the point of view of a sentence user, they're equally simple.

In Chapter 16 we used the pattern matcher's known-values database to introduce the idea of abstract data types. In that example, the most important contribution of the ADT was to isolate the details of the implementation, so that the higher-level procedures could invoke `lookup` and `add` without the clutter of looking for exclamation points. We did hint, though, that the ADT represents a shift in how the programmer thinks about the sentences that are used to represent databases; we don't take the acronym of a database, even though the database *is* a sentence and so it would be possible to apply the `acronym` procedure to it. Now, in thinking about sentences, this idea of shift in viewpoint is more central. Although sentences are represented as lists, they behave much like words, which are represented quite differently.* Our sentence mechanism highlights the *uses* of sentences, rather than the implementation.

* We implemented words by combining three data types that are primitive in Scheme: strings, symbols, and numbers.

Higher-Order Functions

The higher-order functions that we've used until now work only for words and sentences. But the *idea* of higher-order functions applies perfectly well to structured lists. The official list versions of `every`, `keep`, and `accumulate` are called `map`, `filter`, and `reduce`.

`Map` takes two arguments, a function and a list, and returns a list containing the result of applying the function to each element of the list.

```
> (map square '(9 8 7 6))
(81 64 49 36)

> (map (lambda (x) (se x x)) '(rocky raccoon))
((ROCKY ROCKY) (RACCOON RACCOON))

> (every (lambda (x) (se x x)) '(rocky raccoon))
(ROCKY ROCKY RACCOON RACCOON)

> (map car '((john lennon) (paul mccartney)
             (george harrison) (ringo starr)))
(JOHN PAUL GEORGE RINGO)

> (map even? '(9 8 7 6))
(#F #T #F #T)

> (map (lambda (x) (word x x)) 'rain)
ERROR -- INVALID ARGUMENT TO MAP: RAIN
```

The word "map" may seem strange for this function, but it comes from the mathematical study of functions, in which they talk about a *mapping* of the domain into the range. In this terminology, one talks about "mapping a function over a set" (a set of argument values, that is), and Lispians have taken over the same vocabulary, except that we talk about mapping over lists instead of mapping over sets. In any case, `map` is a genuine Scheme primitive, so it's the official grownup way to talk about an `every`-like higher-order function, and you'd better learn to like it.

`Filter` also takes a function and a list as arguments; it returns a list containing only those elements of the argument list for which the function returns a true value. This is the same as `keep`, except that the elements of the argument list may be sublists, and their structure is preserved in the result.

```
> (filter (lambda (flavor) (member? 'swirl flavor))
          '((rum raisin) (root beer swirl) (rocky road) (fudge swirl)))
((ROOT BEER SWIRL) (FUDGE SWIRL))

> (filter word? '((ultra chocolate) ginger lychee (raspberry sherbet)))
(GINGER LYCHEE)


> (filter (lambda (nums) (= (car nums) (cadr nums)))
          '((2 3) (4 4) (5 6) (7 8) (9 9)))
((4 4) (9 9))
```

`Filter` probably makes sense to you as a name; the metaphor of the air filter that allows air through but doesn't allow dirt, and so on, evokes something that passes some data and blocks other data. The only problem with the name is that it doesn't tell you whether the elements for which the predicate function returns `#t` are filtered in or filtered out. But you're already used to `keep`, and `filter` works the same way. `Filter` is not a standard Scheme primitive, but it's a universal convention; everyone defines it the same way we do.

`Reduce` is just like `accumulate` except that it works only on lists, not on words. Neither is a built-in Scheme primitive; both names are seen in the literature. (The name "reduce" is official in the languages APL and Common Lisp, which do include this higher-order function as a primitive.)

```
> (reduce * '(4 5 6))
120

> (reduce (lambda (list1 list2) (list (+ (car list1) (car list2))
                                      (+ (cadr list1) (cadr list2))))
          '((1 2) (30 40) (500 600)))
(531 642)
```

Other Primitives for Lists

The `list?` predicate returns `#t` if its argument is a list, `#f` otherwise.

The predicate `equal?`, which we've discussed earlier as applied to words and sentences, also works for structured lists.

The predicate `member?`, which we used in one of the examples above, isn't a true Scheme primitive, but part of the word and sentence package. (You can tell because it "takes apart" a word to look at its letters separately, something that Scheme doesn't ordinarily do.) Scheme does have a `member` primitive without the question mark that's

like `member?` except for two differences: Its second argument must be a list (but can be a structured list); and instead of returning `#t` it returns the portion of the argument list starting with the element equal to the first argument. This will be clearer with an example:

```
> (member 'd '(a b c d e f g))
(D E F G)

> (member 'h '(a b c d e f g))
#F
```

This is the main example in Scheme of the semipredicate idea that we mentioned earlier in passing. It doesn't have a question mark in its name because it returns values other than `#t` and `#f`, but it works as a predicate because any non-`#f` value is considered true.

The only word-and-sentence functions that we haven't already mentioned are `item` and `count`. The list equivalent of `item` is called `list-ref` (short for "reference"); it's different in that it counts items from zero instead of from one and takes its arguments in the other order:

```
> (list-ref '(happiness is a warm gun) 3)
WARM
```

The list equivalent of `count` is called `length`, and it's exactly the same except that it doesn't work on words.

Association Lists

An example earlier in this chapter was about translating from English to French. This involved searching for an entry in a list by comparing the first element of each entry with the information we were looking for. A list of names and corresponding values is called an *association list,* or an *a-list.* The Scheme primitive `assoc` looks up a name in an a-list:

```
> (assoc 'george
         '((john lennon) (paul mccartney)
           (george harrison) (ringo starr)))
(GEORGE HARRISON)

> (assoc 'x '((i 1) (v 5) (x 10) (l 50) (c 100) (d 500) (m 1000)))
(X 10)

> (assoc 'ringo '((mick jagger) (keith richards) (brian jones)
                  (charlie watts) (bill wyman)))
#F
```

```
(define dictionary
  '((window fenetre) (book livre) (computer ordinateur)
    (house maison) (closed ferme) (pate pate) (liver foie)
    (faith foi) (weekend (fin de semaine))
    ((practical joke) attrape) (pal copain)))

(define (translate wd)
  (let ((record (assoc wd dictionary)))
    (if record
        (cadr record)
        '(parlez-vous anglais?))))
```

`Assoc` returns `#f` if it can't find the entry you're looking for in your association list. Our `translate` procedure checks for that possibility before using `cadr` to extract the French translation, which is the second element of an entry.

Functions That Take Variable Numbers of Arguments

In the beginning of this book we told you about some Scheme procedures that can take any number of arguments, but you haven't yet learned how to write such procedures for yourself, because Scheme's mechanism for writing these procedures requires the use of lists.

Here's a procedure that takes one or more numbers as arguments and returns true if these numbers are in increasing order:

```
(define (increasing? number . rest-of-numbers)
  (cond ((null? rest-of-numbers) #t)
        ((> (car rest-of-numbers) number)
         (apply increasing? rest-of-numbers))
        (else #f)))

> (increasing? 4 12 82)
#T

> (increasing? 12 4 82 107)
#F
```

The first novelty to notice in this program is the dot in the first line. In listing the formal parameters of a procedure, you can use a dot just before the last parameter to mean that that parameter (`rest-of-numbers` in this case) represents any number of

arguments, including zero. The value that will be associated with this parameter when the procedure is invoked will be a list whose elements are the actual argument values.

In this example, you must invoke `increasing?` with at least one argument; that argument will be associated with the parameter `number`. If there are no more arguments, `rest-of-numbers` will be the empty list. But if there are more arguments, `rest-of-numbers` will be a list of their values. (In fact, these two cases are the same: `Rest-of-numbers` will be a list of all the remaining arguments, and if there are no such arguments, `rest-of-numbers` is a list with no elements.)

The other novelty in this example is the procedure `apply`. It takes two arguments, a procedure and a list. `Apply` invokes the given procedure with the elements of the given list as its arguments, and returns whatever value the procedure returns. Therefore, the following two expressions are equivalent:

```
(+ 3 4 5)
(apply + '(3 4 5))
```

We use `apply` in `increasing?` because we don't know how many arguments we'll need in its recursive invocation. We can't just say

```
(increasing? rest-of-numbers)
```

because that would give `increasing?` a list as its single argument, and it doesn't take lists as arguments—it takes numbers. We want *the numbers in the list* to be the arguments.

We've used the name `rest-of-numbers` as the formal parameter to suggest "the rest of the arguments," but that's not just an idea we made up. A parameter that follows a dot and therefore represents a variable number of arguments is called a *rest parameter*.

Here's a table showing the values of `number` and `rest-of-numbers` in the recursive invocations of `increasing?` for the example

```
(increasing? 3 5 8 20 6 43 72)
```

`number`	`rest-of-numbers`	
3	(5 8 20 6 43 72)	
5	(8 20 6 43 72)	
8	(20 6 43 72)	
20	(6 43 72)	(returns false at this point)

In the `increasing?` example we've used one formal parameter before the dot, but you may use any number of such parameters, including zero. The number of formal parameters before the dot determines the *minimum* number of arguments that must be used when your procedure is invoked. There can be only one formal parameter *after* the dot.

Recursion on Arbitrary Structured Lists

Let's pretend we've stored this entire book in a gigantic Scheme list structure. It's a list of chapters. Each chapter is a list of sections. Each section is a list of paragraphs. Each paragraph is a list of sentences, which are themselves lists of words.

Now we want to know how many times the word "mathematicians" appears in the book. We could do it the incredibly boring way:

```
(define (appearances-in-book wd book)
  (reduce + (map (lambda (chapter) (appearances-in-chapter wd chapter))
                 book)))

(define (appearances-in-chapter wd chapter)
  (reduce + (map (lambda (section) (appearances-in-section wd section))
                 chapter)))

(define (appearances-in-section wd section)
  (reduce + (map (lambda (paragraph)
                   (appearances-in-paragraph wd paragraph))
                 section)))

(define (appearances-in-paragraph wd paragraph)
  (reduce + (map (lambda (sent) (appearances-in-sentence wd sent))
                 paragraph)))

(define (appearances-in-sentence given-word sent)
  (length (filter (lambda (sent-word) (equal? sent-word given-word))
                  sent)))
```

but that *would* be incredibly boring.

What we're going to do is similar to the reasoning we used in developing the idea of recursion in Chapter 11. There, we wrote a family of procedures named `downup1`, `downup2`, and so on; we then noticed that most of these procedures looked almost identical, and "collapsed" them into a single recursive procedure. In the same spirit,

notice that all the `appearances-in-` procedures are very similar. We can make them even more similar by rewriting the last one:

```
(define (appearances-in-sentence wd sent)
  (reduce + (map (lambda (wd2) (appearances-in-word wd wd2))
                 sent)))

(define (appearances-in-word wd wd2)
  (if (equal? wd wd2) 1 0))
```

Now, just as before, we want to write a single procedure that combines all of these.

What's the base case? Books, chapters, sections, paragraphs, and sentences are all lists of smaller units. It's only when we get down to individual words that we have to do something different:

```
(define (deep-appearances wd structure)
  (if (word? structure)
      (if (equal? structure wd) 1 0)
      (reduce +
              (map (lambda (sublist) (deep-appearances wd sublist))
                   structure))))

> (deep-appearances
   'the
   '(((the man) in ((the) moon)) ate (the) potstickers))
3

> (deep-appearances 'n '(lambda (n) (if (= n 0) 1 (* n (f (- n 1))))))
4

> (deep-appearances 'mathematicians the-book-structure)
7
```

This is quite different from the recursive situations we've seen before. What looks like a recursive call from `deep-appearances` to itself is actually inside an anonymous procedure that will be called *repeatedly* by `map`. `Deep-appearances` doesn't just call itself once in the recursive case; it uses `map` to call itself for each element of `structure`. Each of those calls returns a number; `map` returns a list of those numbers. What we want is the sum of those numbers, and that's what `reduce` will give us.

This explains why `deep-appearances` must accept words as well as lists as the `structure` argument. Consider a case like

```
(deep-appearances 'foo '((a) b))
```

Since `structure` has two elements, `map` will call `deep-appearances` twice. One of these calls uses the list `(a)` as the second argument, but the other call uses the word `b` as the second argument.

Of course, if `structure` is a word, we can't make recursive calls for its elements; that's why words are the base case for this recursion. What should `deep-appearances` return for a word? If it's the word we're looking for, that counts as one appearance. If not, it counts as no appearances.

You're accustomed to seeing the empty list as the base case in a recursive list processing procedure. Also, you're accustomed to thinking of the base case as the end of a *complete* problem; you've gone through all of the elements of a list, and there are no more elements to find. In most problems, there is only one recursive invocation that turns out to be a base case. But in using `deep-appearances`, there are *many* invocations for base cases—one for every word in the list structure. Reaching a base case doesn't mean that we've reached the end of the entire structure! You might want to trace a short example to help you understand the sequence of events.

Although there's no official name for a structure made of lists of lists of ... of lists, there *is* a common convention for naming procedures that deal with these structures; that's why we've called this procedure `deep-appearances`. The word "deep" indicates that this procedure is just like a procedure to look for the number of appearances of a word in a list, except that it looks "all the way down" into the sub-sub-···-sublists instead of just looking at the elements of the top-level list.

This version of `deep-appearances`, in which higher-order procedures are used to deal with the sublists of a list, is a common programming style. But for some problems, there's another way to organize the same basic program without higher-order procedures. This other organization leads to very compact, but rather tricky, programs. It's also a widely used style, so we want you to be able to recognize it.

Here's the idea. We deal with the base case—words—just as before. But for lists we do what we often do in trying to simplify a list problem: We divide the list into its first element (its `car`) and all the rest of its elements (its `cdr`). But in this case, the resulting program is a little tricky. Ordinarily, a recursive program for lists makes a recursive call for the `cdr`, which is a list of the same kind as the whole argument, but does something non-recursive for the `car`, which is just one element of that list. This time, the `car` of the kind of structured list-of-lists we're exploring may itself be a list-of-lists! So we make a recursive call for it, as well:

```
(define (deep-appearances wd structure)
  (cond ((equal? wd structure) 1)              ; base case: desired word
        ((word? structure) 0)                  ; base case: other word
        ((null? structure) 0)                  ; base case: empty list
        (else (+ (deep-appearances wd (car structure))
                 (deep-appearances wd (cdr structure))))))
```

This procedure has two different kinds of base case. The first two `cond` clauses are similar to the base case in the previous version of `deep-appearances`; they deal with a "structure" consisting of a single word. If the structure is the word we're looking for, then the word appears once in it. If the structure is some other word, then the word appears zero times. The third clause is more like the base case of an ordinary list recursion; it deals with an empty list, in which case the word appears zero times in it. (This still may not be the end of the entire structure used as the argument to the top-level invocation, but may instead be merely the end of a sublist within that structure.)

If we reach the `else` clause, then the structure is neither a word nor an empty list. It must, therefore, be a non-empty list, with a `car` and a `cdr`. The number of appearances in the entire structure of the word we're looking for is equal to the number of appearances in the `car` plus the number in the `cdr`.

In `deep-appearances` the desired result is a single number. What if we want to build a new list-of-lists structure? Having used `car` and `cdr` to disassemble a structure, we can use `cons` to build a new one. For example, we'll translate our entire book into Pig Latin:

```
(define (deep-pigl structure)
  (cond ((word? structure) (pigl structure))
        ((null? structure) '())
        (else (cons (deep-pigl (car structure))
                    (deep-pigl (cdr structure))))))

> (deep-pigl '((this is (a structure of (words)) with)
               (a (peculiar) shape)))
((ISTHAY ISAY (AAY UCTURESTRAY OFAY (ORDSWAY)) ITHWAY)
 (AAY (ECULIARPAY) APESHAY))
```

Compare `deep-pigl` with an `every`-pattern list recursion such as `praise` on page 285. Both look like

```
(cons (something (car argument)) (something (cdr argument)))
```

And yet these procedures are profoundly different. `Praise` is a simple left-to-right walk through the elements of a sequence; `deep-pigl` dives in and out of sublists. The difference is a result of the fact that `praise` does one recursive call, for the `cdr`, while `deep-pigl` does two, for the `car` as well as the `cdr`. The pattern exhibited by `deep-pigl` is called `car-cdr` recursion. (Another name for it is "tree recursion," for a reason we'll see in the next chapter.)

Pitfalls

⇒ Just as we mentioned about the names `word` and `sentence`, resist the temptation to use `list` as a formal parameter. We use `lst` instead, but other alternatives are capital `L` or `seq` (for "sequence").

⇒ The list constructor `cons` does not treat its two arguments equivalently. The second one must be the list you're trying to extend. There is no equally easy way to extend a list on the right (although you can put the new element into a one-element list and use `append`). If you get the arguments backward, you're likely to get funny-looking results that aren't lists, such as

```
((3 . 2) . 1)
```

The result you get when you `cons` onto something that isn't a list is called a *pair*. It's sometimes called a "dotted pair" because of what it looks like when printed:

```
> (cons 'a 'b)
(A . B)
```

It's just the printed representation that's dotted, however; the dot isn't part of the pair any more than the parentheses around a list are elements of the list. Lists are made of pairs; that's why `cons` can construct lists. But we're not going to talk about any pairs that *aren't* part of lists, so you don't have to think about them at all, except to know that if dots appear in your results you're `cons`ing backward.

⇒ Don't get confused between lists and sentences. Sentences have no internal structure; the good aspect of this is that it's hard to make mistakes about building the structure, but the bad aspect is that you might need such a structure. You can have lists whose elements are sentences, but it's confusing if you think of the same structure sometimes as a list and sometimes as a sentence.

⇒ In reading someone else's program, it's easy not to notice that a procedure is making two recursive calls instead of just one. If you notice only the recursive call for the `cdr`, you might think you're looking at a sequential recursion.

⇒ If you're writing a procedure whose argument is a list-of-lists, it may feel funny to let it also accept a word as the argument value. People therefore sometimes insist on a list as the argument, leading to an overly complicated base case. If your base case test says

```
(word? (car structure))
```

then think about whether you'd have a better-organized program if the base case were

```
(word? structure)
```

⇒ Remember that in a deep-structure recursion you may need two base cases, one for reaching an element that isn't a sublist, and the other for an empty list, with no elements at all. (Our `deep-appearances` procedure is an example.) Don't forget the empty-list case.

Boring Exercises

17.1 What will Scheme print in response to each of the following expressions? Try to figure it out in your head before you try it on the computer.

```
> (car '(Rod Chris Colin Hugh Paul))

> (cadr '(Rod Chris Colin Hugh Paul))

> (cdr '(Rod Chris Colin Hugh Paul))

> (car 'Rod)

> (cons '(Rod Argent) '(Chris White))

> (append '(Rod Argent) '(Chris White))

> (list '(Rod Argent) '(Chris White))

> (caadr '((Rod Argent) (Chris White)
           (Colin Blunstone) (Hugh Grundy) (Paul Atkinson)))
```

```
> (assoc 'Colin '((Rod Argent) (Chris White)
                  (Colin Blunstone) (Hugh Grundy) (Paul Atkinson)))

> (assoc 'Argent '((Rod Argent) (Chris White)
                   (Colin Blunstone) (Hugh Grundy) (Paul Atkinson)))
```

17.2 For each of the following examples, write a procedure of two arguments that, when applied to the sample arguments, returns the sample result. Your procedures may not include any quoted data.

```
> (f1 '(a b c) '(d e f))
((B C D))

> (f2 '(a b c) '(d e f))
((B C) E)

> (f3 '(a b c) '(d e f))
(A B C A B C)

> (f4 '(a b c) '(d e f))
((A D) (B C E F))
```

17.3 Describe the value returned by this invocation of `map`:

```
> (map (lambda (x) (lambda (y) (+ x y))) '(1 2 3 4))
```

Real Exercises

17.4 Describe the result of calling the following procedure with a list as its argument. (See if you can figure it out before you try it.)

```
(define (mystery lst)
  (mystery-helper lst '()))

(define (mystery-helper lst other)
  (if (null? lst)
      other
      (mystery-helper (cdr lst) (cons (car lst) other))))
```

17.5 Here's a procedure that takes two numbers as arguments and returns whichever number is larger:

```
(define (max2 a b)
  (if (> b a) b a))
```

Use `max2` to implement `max`, a procedure that takes one or more numeric arguments and returns the largest of them.

17.6 Implement `append` using `car`, `cdr`, and `cons`. (Note: The built-in `append` can take any number of arguments. First write a version that accepts only two arguments. Then, optionally, try to write a version that takes any number.)

17.7 `Append` may remind you of `sentence`. They're similar, except that `append` works only with lists as arguments, whereas `sentence` will accept words as well as lists. Implement `sentence` using `append`. (Note: The built-in `sentence` can take any number of arguments. First write a version that accepts only two arguments. Then, optionally, try to write a version that takes any number. Also, you don't have to worry about the error checking that the real `sentence` does.)

17.8 Write `member`.

17.9 Write `list-ref`.

17.10 Write `length`.

17.11 Write `before-in-list?`, which takes a list and two elements of the list. It should return `#t` if the second argument appears in the list argument before the third argument:

```
> (before-in-list? '(back in the ussr) 'in 'ussr)
#T

> (before-in-list? '(back in the ussr) 'the 'back)
#F
```

The procedure should also return `#f` if either of the supposed elements doesn't appear at all.

17.12 Write a procedure called `flatten` that takes as its argument a list, possibly including sublists, but whose ultimate building blocks are words (not Booleans or procedures). It should return a sentence containing all the words of the list, in the order in which they appear in the original:

```
> (flatten '(((a b) c (d e)) (f g) ((((h))) (i j) k)))
(A B C D E F G H I J K)
```

17.13 Here is a procedure that counts the number of words anywhere within a structured list:

```
(define (deep-count lst)
  (cond ((null? lst) 0)
        ((word? (car lst)) (+ 1 (deep-count (cdr lst))))
        (else (+ (deep-count (car lst))
                 (deep-count (cdr lst))))))
```

Although this procedure works, it's more complicated than necessary. Simplify it.

17.14 Write a procedure `branch` that takes as arguments a list of numbers and a nested list structure. It should be the list-of-lists equivalent of `item`, like this:

```
> (branch '(3) '((a b) (c d) (e f) (g h)))
(E F)

> (branch '(3 2) '((a b) (c d) (e f) (g h)))
F

> (branch '(2 3 1 2) '((a b) ((c d) (e f) ((g h) (i j)) k) (l m)))
H
```

In the last example above, the second element of the list is

```
((C D) (E F) ((G H) (I J)) K)
```

The third element of that smaller list is `((G H) (I J))`; the first element of that is `(G H)`; and the second element of *that* is just `H`.

17.15 Modify the pattern matcher to represent the `known-values` database as a list of two-element lists, as we suggested at the beginning of this chapter.

17.16 Write a predicate `valid-infix?` that takes a list as argument and returns `#t` if and only if the list is a legitimate infix arithmetic expression (alternating operands and operators, with parentheses—that is, sublists—allowed for grouping).

```
> (valid-infix? '(4 + 3 * (5 - 2)))
#T

> (valid-infix? '(4 + 3 * (5 2)))
#F
```